

QUANTUM ENTROPY AND UNCERTAINTY FOR TWO-MODE SQUEEZED, COHERENT AND INTELLIGENT SPIN STATES.

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Abstract

We compute the quantum entropy for monomode and two-mode systems set in squeezed states. Thereafter it is also calculated the quantum entropy for angular momentum algebra when the system is either in a coherent or in an intelligent spin state. These values are compared with the corresponding values of the respective uncertainties. In general, quantum entropies and uncertainties have the same minimum and maximum points. However for coherent and intelligent spin state it is found that some minima for the quantum entropy turn out to be uncertainty maxima. We feel that the quantum entropy we use provide the right answer since it is given in an essentially unique way.

1 INTRODUCTION

Some years ago Deutsch [1] proposed a new definition for the quantum uncertainty of a physical observable which immediately was taken up by Partovi [2] to carefully analyze the measurement of the system (x,p) . Time ago, trying to understand the physical properties of supercoherent states [3] we started to call this new quantity $S(\Phi, |\psi\rangle) \cong -|\langle\psi|\varphi\rangle|^2 \ln |\langle\psi|\varphi\rangle|^2$ the quantum entropy of the system Φ in the state $|\psi\rangle$. In this article we keep using this notation which we feel is more appropriate. There will not be any sort of ambiguity with the standard use of the density operator for the statistical quantum entropy since in the following calculations we will only deal with pure states.

Our motivation is to go further with the quantum entropy and to calculate its values for physical systems less trivial than the monomode (x,p) one. We take the two-mode system when it is set on a two-mode squeezed state and the non-canonical, finite, angular momentum algebra of observables when it is either in a coherent (CSS) or in an intelligent spin state (ISS). Thereafter

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we estimate the (Heisenberg like) uncertainties I of these different systems and compare them with those previously obtained for the quantum entropy. Roughly speaking $S \cong \ln I$ implying coincidence of their extremals. It will be seen that, however, for ISS, it happens that states which minimize the quantum entropy are local maximums for corresponding uncertainty.

In the next section we study the continuous, canonical cases of monomodal and two-modes systems. The third section deal with the three-dimensional angular momentum algebra. In the last one we discuss the results we have obtained.

The problem we are interested in is to compute the quantum entropy $S(\Phi, |\psi\rangle)$ of different physical systems $\Phi = \{(x, p); (x_{\pm}, p_{\pm}); (J_i)\}$ and if possible to determine the states for which $S(\Phi, |\psi\rangle)$ attains its minimum. In this work we do not solve this problem in its full generality. We calculate $S(\Phi, |\psi\rangle)$ for some subspaces of $|\psi\rangle$ and we find the states belonging to these subspaces for which S is extreme. We take the opportunity to compare with uncertainty functionals naturally related to these systems. It is worth pointing out what is the origin of the states we consider: all of them arise through the Heisenberg relations, either by minimizing uncertainty functionals $I(A, B, |\psi\rangle)$ or by introducing intelligent states, i.e. those states which satisfy the functional equation $I(A, B, |\psi\rangle) = C([A, B], |\psi\rangle)$ (C is given below).

2 ONE AND TWO MODE SQUEEZED STATES

This is the continuous and canonical case. We start considering the monomodal case, where $\Phi_1 = \{x, p\}$ and the states $|\psi\rangle$ for which we calculate $S(\Phi_1, |\psi\rangle)$ are the squeezed states [4] (SS) (note the SS arise from the Heisenberg uncertainty relation). If we denote by $|z\rangle$ the standard coherent states

$$|z\rangle \equiv D(a, z)|0\rangle, \quad D(a, z) \equiv \exp\{za^\dagger - z^*a\}, \quad (1a, 1b)$$

the SS are defined

$$|z, r\varphi\rangle \equiv S_1(r, \varphi)|z\rangle, \quad (1c)$$

$$S_1(r, \varphi) \equiv \exp\left\{\frac{1}{2}r(e^{-2i\varphi}a^2 - e^{2i\varphi}(a^\dagger)^2)\right\}. \quad (1d)$$

If one introduces the squeezed annihilation operator $a(r, \varphi)$

$$a(r, \varphi) \equiv S_1(r, \varphi)aS_1^\dagger(r, \varphi) = \cosh r \ a + e^{2i\varphi} \sinh r \ a^\dagger, \quad (2a)$$

the SS turn out to be their eigenvector with eigenvalues z ,

$$a(r, \varphi)|z, r\varphi\rangle = z|z, r\varphi\rangle. \quad (2b)$$

Following Deutsch the quantum entropy $S(\Phi_1, |\psi\rangle)$ is defined by

$$S(\Phi_1, |\psi\rangle) \equiv S(x, |z, r\varphi\rangle) + S(p, |z, r\varphi\rangle) \quad (3a)$$

where

$$S(x, |z, r\varphi\rangle) \equiv - \int_{-\infty}^{\infty} |\langle x|z, r\varphi\rangle|^2 \ln |\langle x|z, r\varphi\rangle|^2 dx$$

$$= 2^{-1}(1 + \ln\{\pi(Re\gamma)^{-1}\}), \quad (3b)$$

$$\begin{aligned} S(p, |z, r\varphi\rangle) &\equiv - \int_{-\infty}^{\infty} | \langle p|z, r\varphi\rangle |^2 \ln | \langle p|z, r\varphi\rangle |^2 dp \\ &= 2^{-1}(1 + \ln\{\pi(Re\gamma)^{-1}\}), \end{aligned} \quad (3c)$$

where

$$\gamma \equiv (\cosh r + e^{2i\varphi} \sinh r)(\cosh r - \sinh r e^{2i\varphi})^{-1}, \quad \gamma^{-1} = \gamma(-r, \varphi). \quad (3d)$$

Consequently S has the value

$$S(\Phi_1, |\psi\rangle) = 1 + \ln \pi + 2^{-1} \ln\{1 + \sinh^2 2r \sin^2 2\varphi\}. \quad (4)$$

$S(\Phi_1, |z, r\varphi\rangle)$ attains its minimum $1 + \ln \pi$ in two cases: i. if $r=0$, i.e. for pure coherent states since $|z, r\varphi\rangle = |z\rangle$ or ii. if $\varphi = n\pi/2$, $n \in \mathbb{Z}$ which corresponds to the proper SS. In both cases one gets what it has been shown [5] to be the minimum of the quantum entropy for this simple system.

To obtain the final results is eqs. (4a-b) we have used that [6]

$$\begin{aligned} \psi\{|z, r\varphi\rangle\}(x) &\equiv \langle x|z, r\varphi\rangle = \\ &= \pi^{-\frac{1}{4}}(Re\gamma)^{\frac{1}{4}} \exp(-i2^{-\frac{1}{2}}Im z) \exp(i2^{\frac{1}{2}}Im z) \exp\{-\gamma 2^{-1}(x - 2^{\frac{1}{2}}Rez)^2\} \end{aligned} \quad (5a)$$

$$\begin{aligned} \bar{\psi}\{|z, r\varphi\rangle\}(p) &\equiv \langle p|z, r\varphi\rangle \\ &= \pi^{-\frac{1}{4}}(Re\gamma^{-1})^{\frac{1}{4}} \exp(i2^{-\frac{1}{2}}Im z) \exp(-i2^{\frac{1}{2}}Im z) \exp\{-(2\gamma)^{-1}(p - 2^{\frac{1}{2}}Im z)^2\}. \end{aligned} \quad (5b)$$

The two-mode system $\Phi_2 = \{x_i, p_i, i \in (1, 2)\}$ has two annihilation and two creation operators a_i, a_i^\dagger ,

$$[a_i, a_i^\dagger] = \delta_{ij}, \quad [a_i, a_j] = 0 \quad (6)$$

The two-mode coherent states are defined by

$$|z\rangle \equiv D(a, z)|0\rangle \equiv (D(a_1, z_1) \otimes D(a_2, z_2))|0\rangle \otimes |0\rangle \quad (7a)$$

where in an obvious two-dimensional vector notation

$$a|z\rangle = z|z\rangle, \quad a \equiv (a_1 \otimes 1_{2 \times 2}, 1_{2 \times 2} \otimes a_2) \quad (7b)$$

The two-mode SS are given by

$$|z, r\varphi\rangle \equiv S_2(r, \varphi)|z\rangle, \quad (8)$$

$$S_2(r, \varphi) \equiv \exp\{r(e^{-2i\varphi}a_1a_2 - e^{2i\varphi}a_1^\dagger a_2^\dagger)\}.$$

Observe that $S_2(r, \varphi)$ contains (a_1, a_1^\dagger) corresponding with two modes we have now in the system. It is possible to generalize eqs. (2) to

$$a(r, \varphi) \equiv S_2(r, \varphi)aS_2^\dagger(r, \varphi) = \cosh r \ a + e^{2i\varphi} \sinh r \sigma_1 a^\dagger, \quad (9a)$$

$$\mathbf{a}(r, \varphi)|\mathbf{z}, r\varphi\rangle = \mathbf{z}|z, r\varphi\rangle. \quad (9b)$$

these eqs. are based upon

$$S_2^\dagger(r, \varphi)\mathbf{D}(\mathbf{a}, \mathbf{z})S_2(r, \varphi) = \mathbf{D}(\mathbf{a}, M(r, \varphi)\mathbf{z}) \equiv \mathbf{D}(\mathbf{a}, \mathbf{w}), \quad (10a)$$

where $M(r, \varphi)$ is defined by

$$\mathbf{w} \equiv M(r, \varphi)\mathbf{z} \equiv \cosh r \mathbf{z} - e^{2i\varphi} \sinh r \sigma_1 \mathbf{z} = (\alpha, \beta)^T \quad (10b)$$

(σ_1 is the standard antidiagonal Pauli matrix). Computation of $S(\Phi_2, |\mathbf{z}, r\varphi\rangle)$ entails the evaluation of $S(\mathbf{x}, |\mathbf{z}, r\varphi\rangle)$ and $S(\mathbf{p}, |\mathbf{z}, r\varphi\rangle)$,

$$\begin{aligned} S(\Phi_2, |\mathbf{z}, r\varphi\rangle) &= S(\mathbf{x}, |\mathbf{z}, r\varphi\rangle) + S(\mathbf{p}, |\mathbf{z}, r\varphi\rangle) \\ &= - \int |\langle \mathbf{x}|\mathbf{z}, r\varphi\rangle|^2 \ln |\langle \mathbf{x}|\mathbf{z}, r\varphi\rangle|^2 d^2x \\ &\quad - \int |\langle \mathbf{p}|\mathbf{z}, r\varphi\rangle|^2 \ln |\langle \mathbf{p}|\mathbf{z}, r\varphi\rangle|^2 d^2p \\ &= 2\{1 + \ln \pi + 2^{-1} \ln \{1 + \sinh^2 2r \sin^2 2\varphi\}\} = 2S(\Phi_1, |\mathbf{z}, r\varphi\rangle). \end{aligned} \quad (11)$$

As it happened for the monomodal case, there are two cases where the entropy has a minimum: i. for $r = 0$, which corresponds to two-mode coherent states and ii. for $\varphi = n\pi/2$, these are the proper two-mode squeezed states. Calculation of $S(\Phi_2, |\mathbf{z}, r\varphi\rangle)$ (11) becomes straightforward after deducing the two (dual) representation of the wave functions $\langle \mathbf{x}|\mathbf{z}, r\varphi\rangle$, $\langle \mathbf{p}|\mathbf{z}, r\varphi\rangle$. It can be seen that

$$\begin{aligned} |\langle \mathbf{x}|\mathbf{z}, r\varphi\rangle|^2 &= \pi^{-1} \operatorname{Re} \alpha \{1 - (\operatorname{Re} \beta)^2 (\operatorname{Re} \alpha)^{-2}\}^{\frac{1}{2}} \\ &\cdot \exp\{-\operatorname{Re} \alpha (\mathbf{x} - 2^{\frac{1}{2}} \operatorname{Re} \mathbf{w})^2 - \operatorname{Re} \beta (\mathbf{x} - 2^{\frac{1}{2}} \operatorname{Re} \mathbf{w})^T \sigma_1 (\mathbf{x} - 2^{\frac{1}{2}} \operatorname{Re} \mathbf{w})\} \end{aligned} \quad (12a)$$

$$\begin{aligned} |\langle \mathbf{p}|\mathbf{z}, r\varphi\rangle|^2 &= \pi^{-1} \operatorname{Re} \alpha \{1 - (\operatorname{Re} \beta)^2 (\operatorname{Re} \alpha)^2\}^{\frac{1}{2}} \\ &\cdot \exp\{-\operatorname{Re} \alpha (\mathbf{p} - 2^{\frac{1}{2}} \operatorname{Im} \mathbf{w})^2 - \operatorname{Re} \beta (\mathbf{p} - 2^{\frac{1}{2}} \operatorname{Im} \mathbf{w})^T \sigma_1 (\mathbf{p} - 2^{\frac{1}{2}} \operatorname{Im} \mathbf{w})\} \end{aligned} \quad (12b)$$

The uncertainty $I(\mathbf{x}, \mathbf{p}, |\mathbf{z}, r\varphi\rangle)$ for the monomodal case is the standard quantity $(\Delta x)^2(|\mathbf{z}, r\varphi\rangle)$ $(\Delta p)^2(|\mathbf{z}, r\varphi\rangle)$. It turns out to be

$$I(\Phi_1, |\mathbf{z}, r\varphi\rangle) = 4^{-1}(1 + \sinh^2 2r \sin^2 \varphi)$$

Consequently, since $S(\Phi_2, |\mathbf{z}, r\varphi\rangle) \cong C_1 + 2^{-1} \ln I(\Phi_1, |\mathbf{z}, r\varphi\rangle)$ we observe that their minima coincide. For the two-mode system one has the uncertainty matrix [6] $I(\Phi_2, |\mathbf{z}, r\varphi\rangle)$ defined by

$$\begin{aligned} I(\Phi_2, |\mathbf{z}, r\varphi\rangle) &\equiv (\Delta \mathbf{x})^2 (\Delta \mathbf{p})^2 = \\ &\begin{pmatrix} (\Delta x_1)^2 & (\Delta x_1)(\Delta x_2) \\ (\Delta x_1)(\Delta x_2) & (\Delta x_2)^2 \end{pmatrix} \begin{pmatrix} (\Delta p_1)^2 & (\Delta p_1)(\Delta p_2) \\ (\Delta p_1)(\Delta p_2) & (\Delta p_2)^2 \end{pmatrix} \end{aligned} \quad (14a)$$

Minimum uncertainty states (MUS) are defined as those for which $I(\Phi_2, |\psi\rangle) \cong 1_{2 \times 2}/4$. In the present case we have that

$$I(\Phi_2, |\mathbf{z}, r\varphi\rangle) = 4^{-1}(1 + \sinh^2 2r \sin^2 \varphi) \cdot 1_{2 \times 2}.$$

We have the qualitative situation already discussed for the monomodal case: minima of $S(\Phi_2)$ and $I(\Phi_2)$ coincide with either two-mode coherent or with proper squeezed states. Now we shift our interest to consider the less traditional, finite, non canonical system generated by 3-dimensional angular momentum algebra.

3 THE ANGULAR MOMENTUM ALGEBRA, COHERENT AND INTELLIGENT SPIN STATES.

The three-dimensional angular momentum algebra Φ_J provide a simple example of what one might think to be a general physical system. Its three generators J_i , $i \in (1, 2, 3)$ satisfy the commutation relations

$$[J_i, J_j] = i\epsilon_{ijl}J_l. \quad (15)$$

In general we will think of a physical system Φ_A to be a set of observables $\Phi_A = \{A_i, i \in \omega_p\}$ constituting some algebraic structure (i.e. very often this structure is a Lie algebra). The natural generalization of the quantum entropy definition initially given [1] for Φ_1 (eqs. (3)) is

$$S(\Phi_A, |\psi\rangle) \equiv \sum_{i=0}^{i=p} S(A_i, |\psi\rangle) = - \sum_{i=1}^{i=p} \sum_{\alpha_i} |\langle \alpha_i | \psi \rangle|^2 \ln |\langle \alpha_i | \psi \rangle|^2 \quad (16a)$$

where $|\alpha_i\rangle$ are the eigenstates with eigenvalues α_i of the observable A_i ,

$$A_i |\alpha_i\rangle = \alpha_i |\alpha_i\rangle. \quad (16b)$$

Actually, a physical system Φ might be considered represented by different sets of observables $\{A_i\}$, $\{B_i\}$, \dots which can be thought as equivalent quantum atlases which represent Φ .

In terms of field theory one is thinking in the possibility of $\{B_i\}$ being a redefinition of the initial observables $\{A_i\}$.

An already interesting, and non trivial example is whether, following this definitions of a physical system, $\Phi_1 = \{x, p\}$ can also be represented by $\{N \equiv a^\dagger a, \hat{\phi}\}$, the number and a convenient phase operator [7]. Of course, one expects that the quantum entropy of a physical system Φ must be independent of its quantum representation, $S(\Phi_A, |\psi\rangle) = S(\Phi_B, |\psi\rangle)$. We will not dwell on this interesting point in this article. Entropic calculations will be compared with uncertainties, which do not have a clearly cut, inambiguous definition, as we will comment below.

One of our main motivations of the present calculations is to better understand which are, for each specific given physical system, the states $|\psi\rangle$ minimizing its quantum entropy, i.e. those states satisfying

$$\frac{\delta}{\delta |\psi\rangle} S(\Phi, |\psi\rangle) + \lambda |\psi\rangle = 0, \quad \langle \psi | \psi \rangle = 1. \quad (17a - b)$$

Instead of directly solving this problem, which we cannot do now, we study the behaviour of $S(\Phi, |\psi\rangle)$ for subfamilies $|\psi\rangle$ having a relevant physical origin, related to or stemming in uncertainty relations.

It is worth recalling what is the general situation concerning uncertainties functionals [8]. Given two physical observables A_1, A_2 Schwarz's inequality tell us that, for physical states $|\psi\rangle$: $\langle \psi | \psi \rangle = 1$,

$$I(A_1, A_2, |\psi\rangle) \equiv (\Delta A_1^2)(\Delta A_2^2) \geq 4^{-1} |\langle [A_1, A_2] \rangle|^2 \equiv C([A_1, A_2], |\psi\rangle). \quad (18)$$

MUS (minimum uncertainty states) are those for which I has a local minimum and IS (intelligent states) are states that satisfy the equality in eq. (18). The role of physical theories is to

provide the value of commutator $[A_1, A_2]$. In principle one may find $|\psi_{MUS} \rangle \neq IS$, $|\psi_I \rangle \neq MUS$ and $|\psi_{MUS,I} \rangle$. It seems that IS constitute a very large set, being the states corresponding to intersection of two functionals.

Φ_J has two properties: i. is finite, i.e. it has irreducible unitary representations which are finite (due to compactness of $SO(3)$) and ii. is non canonical, i.e. there are not additional observables $K_j : [J_i, K_j] = i\delta_{ij} \cdot \Phi_J$ is one the simplest physical systems where there are IS which are not MUS [8]. The two kind of states that will be considered here are the coherent (CSS) [9] and the intelligent spin states (ISS) [8].

CSS are given by

$$|CSS \rangle \equiv |\tau \rangle \equiv R(\tau) | -j \rangle = (1 + \tau\tau^*)^{-j} e^{\tau J} | -j \rangle \quad (19a)$$

$$\begin{aligned} R(\tau) &\equiv \exp\{\tau J_+\} \exp\{\ln(1 + \tau\tau^*) J_3\} \exp\{-\tau^* J_-\} \\ &= \exp\{-i\theta J_n\}, \hat{n} = (\sin \varphi, -\cos \varphi, 0) \end{aligned} \quad (19b)$$

where

$$\tau = e^{-i\varphi} \tan\left(\frac{\theta}{2}\right), \quad 0 \leq \theta \leq \pi, \quad 0 \leq \varphi \leq 2\pi. \quad (19c)$$

ISS $|w_{j,n}(\tau) \rangle$ have been defined as those for which $I(J_1, J_2, |w(\tau) \rangle) = C(J_3, |w(\tau) \rangle)$. They turn out to be

$$|w_{j,n}(\tau) \rangle = a_n Y_1 \partial_y^n \{y^{2j} e^{\tau_y J_+} | -j \rangle\}, \quad 0 \leq n \leq 2j \quad (20a)$$

where

$$a_n \equiv \{Z_1 Y_1 \partial_y^n \partial_z^n [yz + \tau\tau^*(y-2)(z-2)]^{2j}\}^{-\frac{1}{2}}, \quad (20b)$$

$$Y_1 F(y, z) \equiv F(1, z), \tau_y \equiv \tau(1 - 2/y), \tau^2 = \tau^{*2} \quad (20c-d)$$

In particular $|w_{j,0}(\tau) \rangle = | -\tau \rangle$ and $|w_{j,2j}(\tau) \rangle = |\tau \rangle$ are CSS. We denote $|m \rangle_i$ the respective eigenstates of J_i ,

$$J_i |m \rangle_i = m |m \rangle_i \quad (21)$$

We first calculate $S(\Phi_J, |\tau \rangle)$. According to eqs. (16)

$$S(\Phi_J, |\tau \rangle) = - \sum_{i=1}^3 \sum_{m=-j}^{m=j} |i \langle m | \tau \rangle|^2 \ln |i \langle m | \tau \rangle|^2. \quad (22a)$$

It is immediate to obtain the values of $i \langle m | \tau \rangle$ and its associated probabilities

$$|_{1,2} \langle m | \tau \rangle|^2 =$$

$$\{2(1 + \tau\tau^*)\}^{-2j} \alpha(j, m) |1, i + \tau|^{2(j+m)} |1, i - \tau|^{2(j-m)}, \quad (22b)$$

$$|_3 \langle m | \tau \rangle|^2 = \{(1 + \tau\tau^*)\}^{-2j} \alpha(j, m) |\tau|^{2(j+m)}, \quad (22c)$$

$$\alpha(j, m) \equiv 2j!(j+m)!/(j-m)!$$

No closed expression has been obtained for eq. (22a). The same happens with the entropy for ISS. Its value is

$$S(\Phi_J, |w_n(\tau) \rangle) = - \sum_{i=1}^3 \sum_{m=-j}^{m=j} |i \langle m | w_n(\tau) \rangle|^2 \ln |i \langle m | w_n(\tau) \rangle|^2 \quad (23)$$

where

$$\begin{aligned} \langle w_n(\tau) | m \rangle_{1,2} &= a_n \alpha(j, m)^{\frac{1}{2}} \sum_{p=0}^{p=j-m} 2^{-m-p} (-1, -i)^p (j+m+p)! (p!(j-m-p)!)^{-1} \\ &\sum_{q=0}^{p=j+m+p} (1, -i)^q (q!(j+m+p-q)!)^{-1} \tau^{j+m+p-q} \rho_n^{j+m+p-q}, \end{aligned} \quad (24a)$$

$$|\langle w_n(\tau) | m \rangle_3|^2 = a_n \alpha(j, m)^{\frac{1}{2}} |\tau|^{2(j+m)} (\rho_n^{j+m})^2, \quad (24b)$$

$$\tau = \tan\left(\frac{\theta}{2}\right) e^{-in\pi/2}, \quad n \in \mathbb{Z}, \quad \rho_n^k \equiv Y_1 \partial_y^n \{y^{2j-k} (y-2)^k\}. \quad (24c)$$

Fig. 1 shows the structure of $S(\Phi_J, |\tau\rangle)$ in terms of the θ, φ parametrization eq. (19c). S has local minimums at $\varphi = n\pi/2$, $\theta = \pi/2$. Details of the θ dependence for $\varphi = n\pi/2$ appear in fig. 2,3. It can be also observed that the minimum values of S increase with j .

Then we present in fig. 4 $S(\Phi_J | w_{j,n}(\tau)\rangle)$ for the first proper ISS $|w_{1,1}(\tau)\rangle$ ($|w_{1,0}(\tau)\rangle$ and $|w_{1,2}(\tau)\rangle$ are CSS), and just to have a better feeling of its behaviour we show, in fig. 5, the shape of $S(\Phi_J, |w_{j,n}(\tau)\rangle)$ for $j = 2$, $n = 1, 3$, proper non coherent intelligent spin states.

Then, fig. 6 shows that the minimum for S occur for the central ISS, i.e. in case of $j = 2$ for $|w_{2,2}(\tau)\rangle$. In general it will occur for $n = (j, j \pm 1/2)$ according to whether j is integer or half-integer.

What can be said about the uncertainties?

In spite of arguments given [10] in favor of $\Delta J \equiv (\Delta J^2)^{1/2}$ as the right quantity one should take to define the uncertainty of Φ_J (ΔJ is a clear rotational invariant quantity), we will take partial and full quadratic uncertainties $I(J_1, J_2, |\psi\rangle)$,

$$I(\Phi_J, |\psi\rangle) = I(J_1, J_2, |\psi\rangle) + I(J_3, J_3, |\psi\rangle) + I(J_3, J_1, |\psi\rangle)$$

as the physical relevant quantities which provide an additional insight concerning informational behaviour of Φ_J .

It seems to us that quadratic uncertainties are the typical elements of a quantum mechanically based definition.

As it is shown in figs. 7,8 there is a sharp qualitative difference in the behaviour of $I(\Phi_J, |\psi\rangle)$ and $I(J_1, J_2, |\psi\rangle)$. While $I(J_1, J_2, |w_n(\tau)\rangle)$ presents a local minimum at $\theta = \pi/2$ ($|\tau| \cong 1$); $I(\Phi_J | w_{j,n}(\tau)\rangle)$ has a local maximum at this same point.

Since $S(\Phi_J, |w_{j,n}(\tau)\rangle)$ exhibits a local minimum at $\theta = \pi/2$ and the full uncertainty shows a maximum, one cannot qualitatively relate anymore these two quantities through $S \cong \ln I$. these property is exhibit in fig. 7 where it is shown the anomalous behaviour of $I(\Phi_J, |w_{j,n}(\tau)\rangle)$. Partial uncertainty for $I(J_1, J_2, |w_{j,n}(\tau)\rangle)$ is shown in fig. 8. Its behaviour is completely different of the full uncertainty. Partial uncertainty minima coincide with entropy minimums.

4 CONCLUSIONS

We have estimated the values of the quantum entropy (not to be confused with the statistical quantum entropy due to the statistical mixture of quantum states) for monomode squeezed and

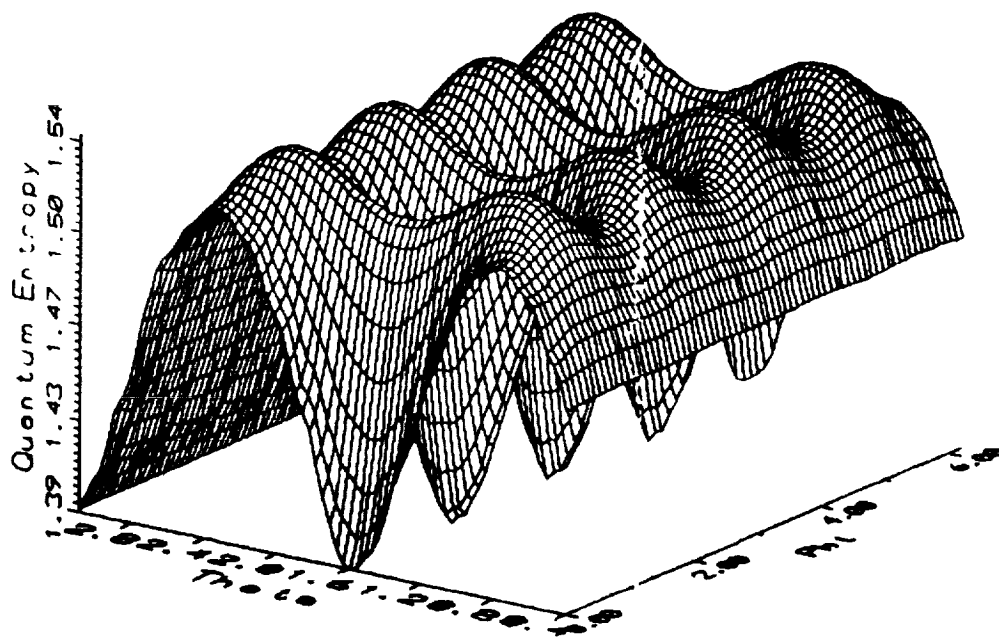


Fig. 1 Quantum entropy for coherent spin states
(CSS : $J = 0.5$)

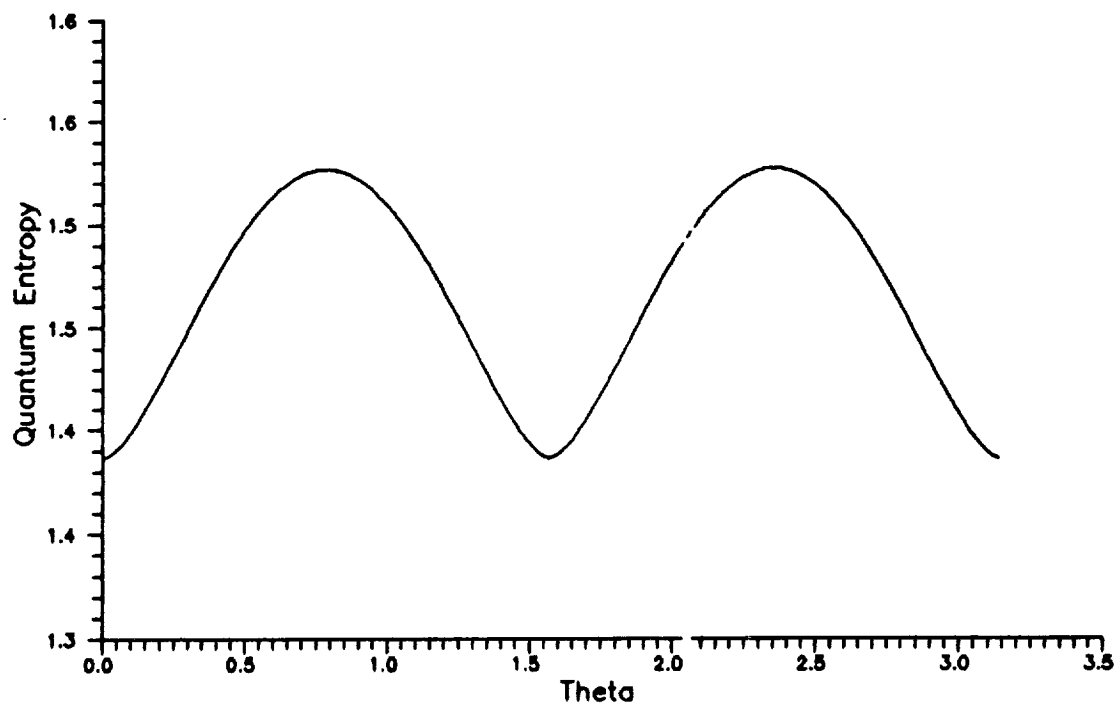


Fig.2 Quantum entropy for coherent spin states (CSS : $j = 1/2$, $\varphi = n\pi / 2$).

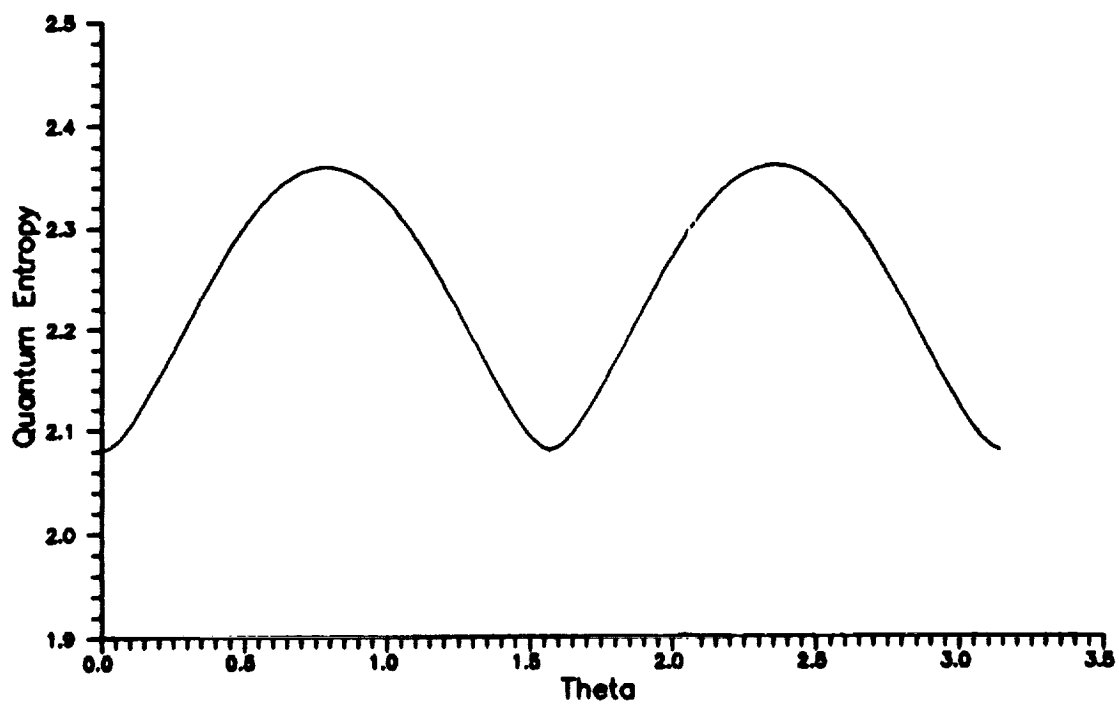


Fig.3 Quantum entropy for coherent spin states (CSS : $j = 1$, $\varphi = n\pi/2$).

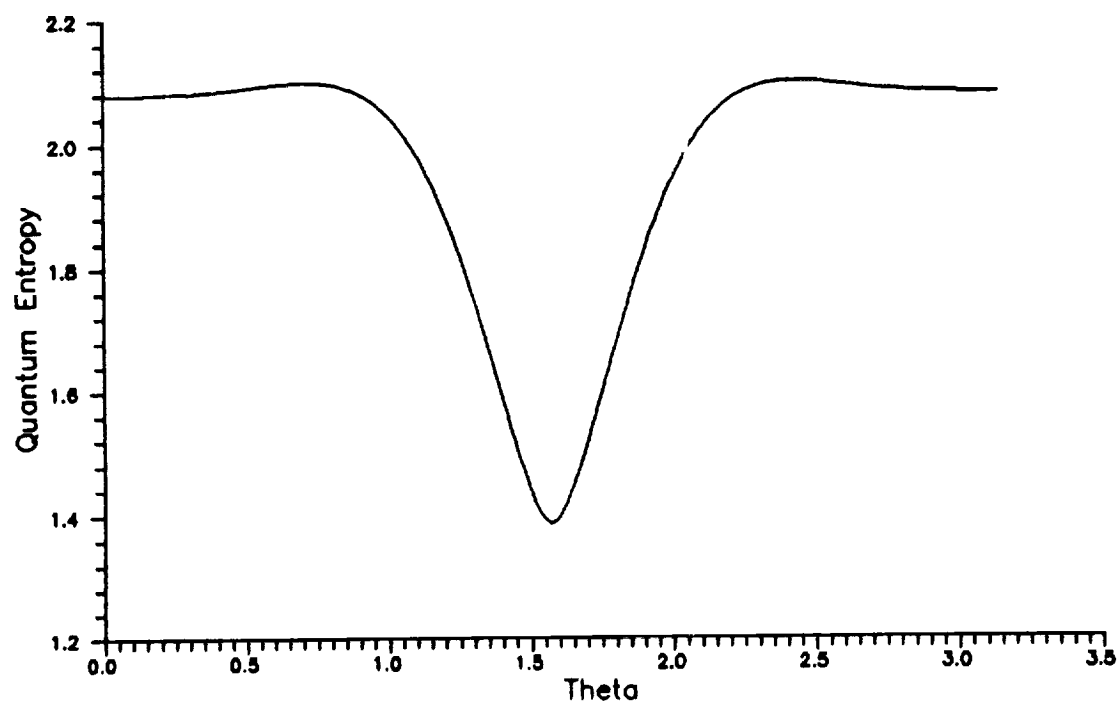


Fig.4 Quantum entropy for intelligent spin states (ISS : $j = 1$, $n = 1$).

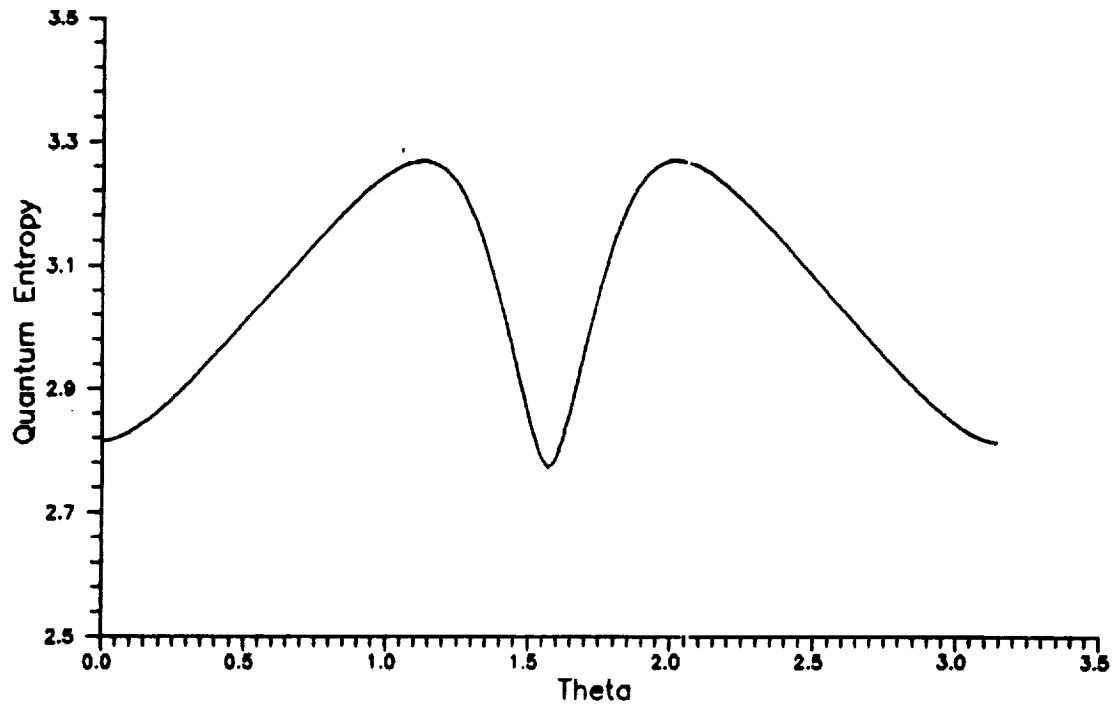


Fig.5 Quantum entropy for intelligent spin states (ISS : $j = 2$, $n = 1,3$).

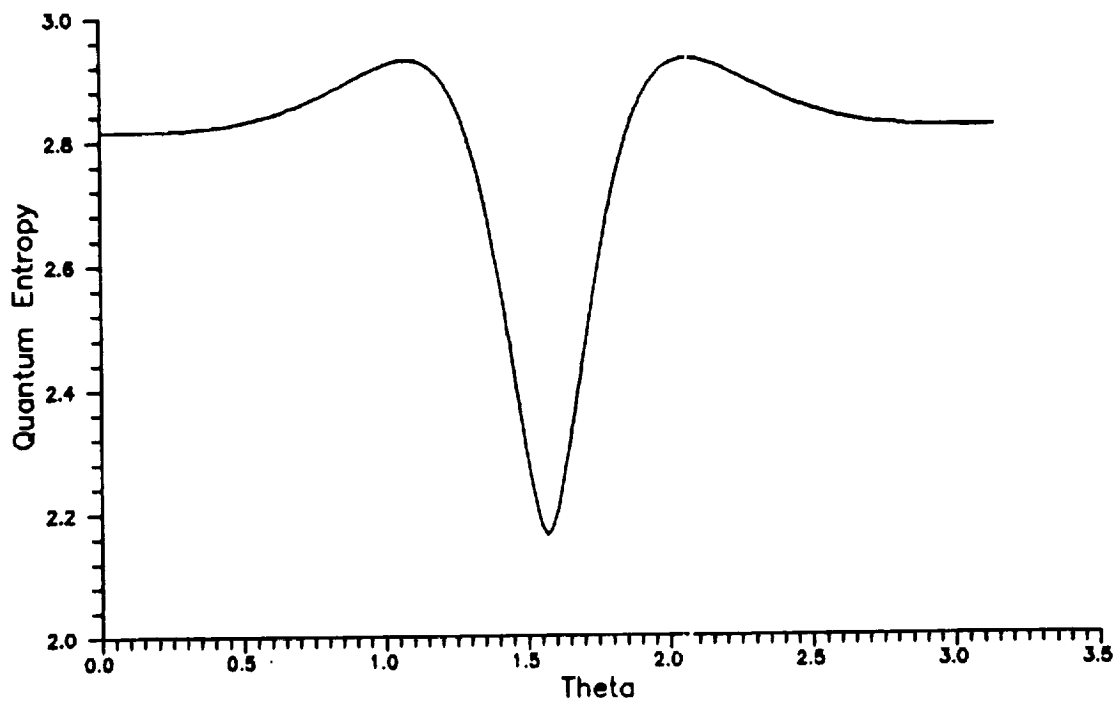


Fig.6 Quantum entropy for intelligent spin states (ISS : $j = 2$, $n = .2$).

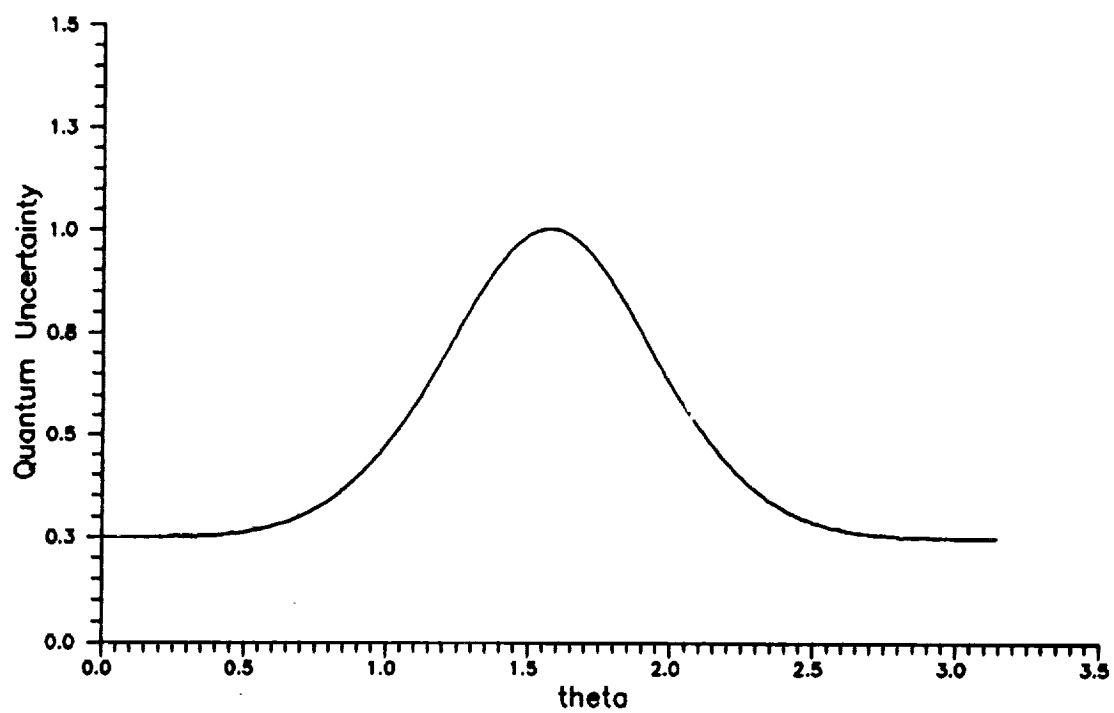


Fig.7 Full quantum uncertainty for intelligent spin state (ISS : $j = 1$, $n = 1$)

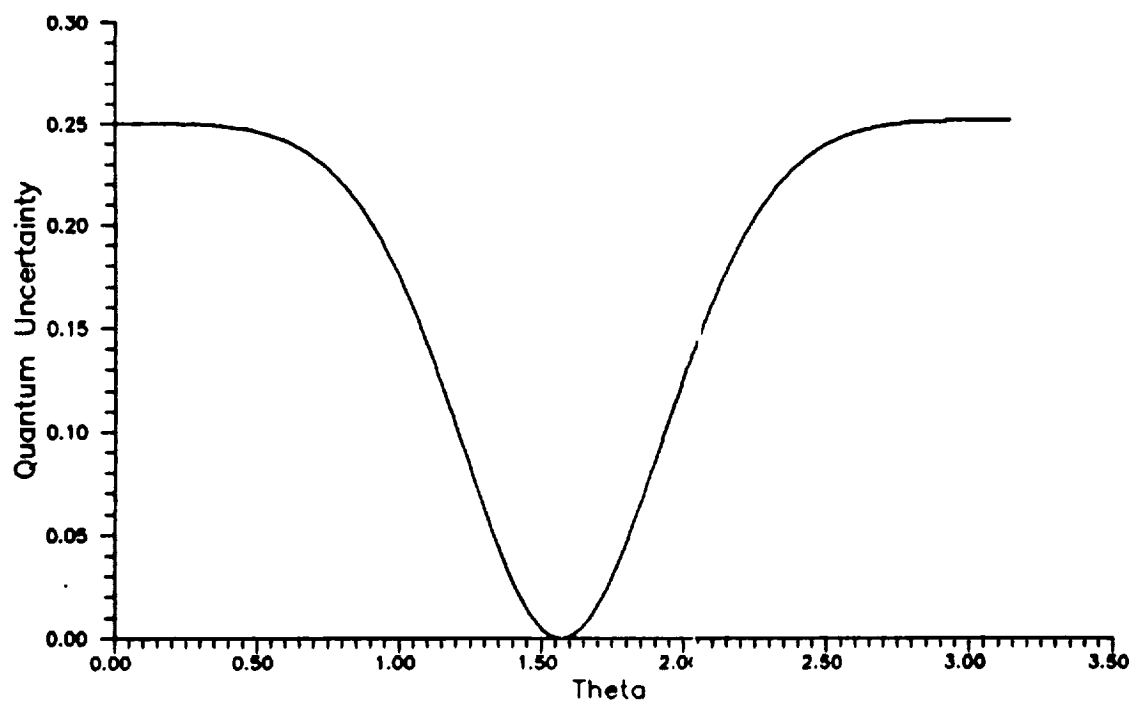


Fig.8 Partial quantum uncertainty for intelligent spin states (ISS : $j = 1$, $n = 1$)

two mode squeezed states. Calculations were extended to the angular momentum system Φ_J where the states we used to probe $S(\Phi_J)$ came from the natural generalization of the standard coherent states or by imposing intelligence, i.e. states which satisfy the now operatorial Heisenberg equality.

In this case, the proper central IS were shown to be the best ones, i.e. they minimize $S(\Phi_J)$. We systematically compared the behaviour of $S(\phi_J)$ with that of $I(\Phi_J)$ just to understand why one must abandon the use of these latter quantities in favor of $S(\Phi_J)$. We observed the presence of anomalous behaviour in $I(\Phi_J)$ when one considers ISS, giving additional support to the choice of $S(\Phi_J)$ as the right physical quantity one has to consider for every physical system.

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References

- [1] Deutsch D., Phys. Rev. Lett. **50** (1983) 631.
- [2] Partovi M. H., Phys. Rev. Lett. **50** (1983) 1883.
- [3] Aragone C. and Zypman, J. Phys. **A19** (1986) 2267.
- [4] Yuen H. P., Phys. Rev. **A13** (1976) 2226.
- [5] Bialynicki-Birula I and Mycielsky J., Comm. Math. Phys. **44** (1975) 129.
- [6] Schumaker B. L., Physics Reports **135** (1986) 317.
- [7] Pegg D. J. and Barnett S. M., Europhysics Letters **6** (1988) 483.
Sussking L. and Glogower, Physics **1** (1964) 49.
- [8] Aragone C., Chalbaud E. and Salamó S., J. of Math. Phys. **17** (1976) 1963.
- [9] Radcliffe J. M., J. Phys **A4** (1971) 313.
Arecchi F. T., Courtens E., Gilmore R. and Thomas H., Phys. Rev. **A6** (1972) 2211.
- [10] Delbourgo R., J. Phys. **A10** (1977) 1837.